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ABSTRACT

For learners to develop powerful conceptions of mathematics, they must have opportunities to experience mathematics--to make conjectures, explore mathematical relationships, justify claims, engage in mathematical communication, and connect concepts within and outside of mathematics. If learners are to experience mathematics in this way, significant implications for teachers, the preparation of beginning teachers in particular, will follow. This paper explores several of these implications as they relate specifically to the content knowledge that prospective mathematics teachers bring to the preparation process. Moreover, it examines a model for mathematics teacher education that provides beginning teachers an opportunity to grapple with the relationships between their own mathematical knowledge and conceptually-based teaching. Following a review of the literature, the paper presents a case study of a mathematics major who had returned to graduate school for secondary teaching licensure. The intent is to use the student's experiences as a springboard for discussions about the potential impact that elementary mathematics content and pedagogy might have on beginning secondary teachers. Contains 21 references. (SAH)

Elementary Mathematics:

A Missing Piece in Secondary Mathematics Teacher Education?

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Elementary Mathematics:

A Missing Piece in Secondary Mathematics Teacher Education?

The subject matter knowledge of prospective mathematics teachers has become a central issue in mathematics education research (Ball, 1990; Ball & McDiarmid, 1990). The reason for this increased attention to mathematics teachers' subject knowledge may be attributed in part to heightened expectations for student learning such as those found in the National Council of Teachers of Mathematics' (NCTM 1989, 1991) *Standards* documents. The *Standards* are representative of other reform initiatives in which the underlying goal of mathematics teaching is for students to develop rich mathematical understandings and "mathematical power" (NCTM, 1989). For learners to develop powerful conceptions of mathematics, they must have opportunities to experience mathematics -- to make conjectures, explore mathematical relationships, justify claims, engage in mathematical communication, and connect concepts within and outside of mathematics. If learners are to experience mathematics in this way, significant implications for teachers, and the preparation of beginning teachers in particular, follow. This article explores several of these implications as they relate specifically to the content knowledge that prospective mathematics teachers bring to the preparation process. Moreover, it examines a model for mathematics teacher education that provides beginning teachers an opportunity to grapple with the relationships between their own mathematical knowledge and conceptually-based teaching.

Conceptual Framework

Mathematics education reform has drawn attention to themes of problem solving, mathematical connections, mathematical modeling, and high level reasoning (NCTM, 1989, 1991). Despite wide support for such curricular and pedagogical reform in school mathematics, however, classroom instruction has been slow to deviate from existing models of teacher-centered, expositional practices (Weiss, 1995). These findings are particularly true of novice secondary teachers who often struggle to reconcile the vision of mathematics reform they encounter in preparation programs with the realities of the school classroom (Frykholm, 1996; Frykholm and

Brendefur, 1997). As explored in the following paragraphs, one factor that certainly contributes to the frequency with which beginning teachers adopt lecture-oriented and teacher-dominated instructional tendencies is mathematical content knowledge.

As Ball (1990) has suggested, teachers must understand mathematics deeply themselves if they are to facilitate the types of discussions and handle the various questions that emerge when learners are engaging in authentic mathematical experiences. As Shulman and Grossman (1988) have noted, subject matter knowledge significantly impacts classroom instruction, as well as teachers' decisions with respect to the selection and structure of teaching content, classroom activities, assignments, and choices in curriculum materials. Brown and Borko (1992) summarized their review of literature on subject matter knowledge by stating that,

> Greater subject matter knowledge enabled teachers to connect topics within a subject and to provide conceptual explanations, as opposed to purely algorithmic ones. In mathematics, participants with greater subject matter knowledge were more likely to see problem solving as central to mathematics instruction and to emphasize a conceptual approach to teaching (p. 217).

As an extension of the previous assertions, Shulman (1986) has suggested that content knowledge, by itself, does not guarantee effective teaching. Novice teachers must also acquire pedagogical content knowledge -- "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Shulman and Grossman (1988) have noted the connections between subject matter knowledge and pedagogical content knowledge as they described teachers for whom "knowledge of subject matter became infused with their knowledge of students, their knowledge of teaching, and their knowledge of curriculum" (p. 19). Such rich knowledge about mathematics and teaching is, of course, a laudable goal. There is considerable evidence however, to suggest that beginning teachers do not have the kinds of content and pedagogical content knowledge discussed in the previous paragraphs (Brown and Borko, 1992).

For example, Frykholm (1996) has suggested that, although beginning teachers report that they value reform-based teaching ideals, they do not have enough experience, content knowledge, and confidence to deviate from primarily lecture-based, rote instruction. Because these beginning teachers often do not possess deep and connected understandings of mathematics themselves, it is difficult for them to create rich learning opportunities for their students such as those promoted in the reform literature. Similarly, Ball (1990) challenged three common assumptions about school mathematics, the knowledge bases of prospective secondary teachers, and the learning-to-teach process. She called into question the notions that: (1) the traditional school mathematics content is not difficult, (2) pre-college education provides teachers with much of what they need to know about mathematics, and (3) majoring in mathematics ensures subject matter knowledge. In her study with over 250 prospective teachers, Ball found that secondary mathematics education candidates believed they knew mathematics, felt confident in their ability to do mathematics, and felt as though mathematics could be explained. In truth, however, they were no more successful than elementary candidates in providing conceptual explanations for mathematical concepts. Rather, they tended to offer mathematical "rules" as explanations for concepts. As Ball (1990) reported, "Almost all the prospective teachers in both [secondary and elementary] groups agreed that remembering rules and facts was essential. The difference was that the secondary candidates remembered these rules better than the elementary majors and used them as explanations. In other words, the rules *were* explanations" (p. 460).

These research findings are critical conceptual underpinnings for the study reported in this paper. My work with secondary pre-service teachers over the years confirms Ball's assertion that many mathematics education students do not possess rich enough mathematical knowledge to promote deep mathematical understanding in the classroom. Prospective secondary teachers typically enter the preparation process with fairly rigid and fixed conceptions of mathematics (Brown and Borko, 1992) that make it difficult for them to envision open classrooms in which multiple solution strategies are encouraged, where children teach each other, and where the teacher relinquishes the role of the mathematical "authority" (Frykholm, in press). Based on these belief systems, beginning teachers often then implement the very kinds of teacher-centered instructional strategies the reform movement (and methods courses) have endeavored to offset.

One implication that follows, therefore, is that teacher educators need to find opportunities in methodology courses to "broaden, and in many cases challenge, the belief and knowledge structures of...future mathematics teachers" (Frykholm, 1996, p. 679). This call is certainly consistent with other scholars in mathematics education who also have noted the link between beginning teachers' beliefs, knowledge, and eventual classroom practices (see Thompson, 1992; Putnam, Lampert & Peterson, 1990). This article further examines these relationships between novice teachers' knowledge and dispositions toward teaching.

Research Context

Building on the notion that secondary pre-service teachers often do not have rich conceptual understandings of fundamental k-12 mathematical concepts, the research reported in this article pursued the possibility of integrating aspects of secondary and elementary mathematics teacher preparation in order to more fully prepare novice secondary teachers to teach for understanding. Driving the study was the idea that the rigid conceptions of mathematics and mathematics instruction often held by prospective secondary teachers might best be challenged by encouraging them to explore and think about mathematics in a way that a young child might -- by manipulating objects, exploring spatial relationships, and experiencing mathematics through the context of daily routines and activities. As the findings presented in this article suggest, engaging substantively in mathematical content and pedagogical issues related to elementary level mathematics education may lead to a richer, more substantial preparation experience for prospective secondary teachers.

Methodology

Overview

The bulk of this article is a case study of Julie, a mathematics major who had returned to graduate school for secondary teaching licensure. In organizing her plan of study at the beginning of her graduate program, Julie was interested in courses that would have the most immediate impact on her development as a mathematics teacher. One of her options was to take a course that examined elementary mathematics content and pedagogy. Julie became interested in the idea, struck by the possibilities this course held for enhancing her development as a secondary teacher.

This study documents Julie's experiences as she progressed through the secondary teacher preparation process, and, specifically, engaged in this course on elementary mathematics.

The primary intent of this article is to use Julie's experiences as a springboard for discussions about the potential impact that elementary mathematics content and pedagogy might have on beginning secondary teachers. As such, in addition to presenting Julie's case, this article also examines excerpts from data collected from Julie's cohort of prospective secondary mathematics teachers. Of special interest were their impressions of elementary level mathematics, their ability to explain elementary-level mathematics topics conceptually, and how they felt elementary methods and concepts might potentially impact their teaching at the secondary level. Again, this data is presented to provide a context for a discussion on how Julie's experiences with elementary mathematics might address some of the general challenges facing secondary teacher preparation in mathematics.

Subjects

Julie was one of a cohort of 26 students in a Master's level secondary mathematics education program. This cohort took two secondary-level mathematics methods courses together (including two field experiences), as well as a number of other general education courses over a two year span of time.

Case study design

A case study design (Yin, 1984) was adopted to examine Julie's experiences within the secondary teacher education program. As Adelman, Jenkins and Kemmis (1976) have suggested, a case is a "bounded system" that represents "a detailed examination of one setting, or one single subject, or one single depository of documents, or one particular event" (Bogdan and Biklen, p. 58). As such, Julie's progression through the preparation process, and in particular her evolving beliefs about conceptual teaching, were examined closely within the context of the secondary teacher preparation program. As Yin (1984) has suggested, the goal of the investigator in case study research is to "to expand and generalize theories" (p. 21). Grossman (1990) has added that gathering rich and detailed data "contribut[es] to a broader conceptualization of teacher knowledge and its use in teaching" (p. 150). Hence, Julie's case is intended to shed light on a possible innovative construct in secondary mathematics teacher preparation -- the blending of elementary and secondary methodology courses within the preparation experience.

Data sources

For the cohort of prospective secondary teachers, data sources included: discussions that took place within the context of the methods courses, students' work on a series of classroom problems and activities, students' free-write responses to various writing prompts, journal responses, and four email listserv discussions throughout the year that focused on k-12 mathematics. For Julie's case, additional data sources were gathered, guided by Bullough, Knowles and Crow's (1991) suggestion that "case study methodology...is a responsive methodology, avoiding rigid data-gathering strategies, just as it avoids interpretations made in advance of data gathering" (p. 12). As such, data was gathered informally throughout the three semester study as Julie interacted throughout the study with her students and her peers in a variety of contexts. Specific opportunities to collect further data from Julie included six classroom observations (while student teaching), six post-lesson conferences, free-write responses, and two interviews before and after the elementary methods course.

Data analysis

Data analysis was influenced by models of qualitative research advocating a systematic and ongoing fracturing of the data, leading to an identification of core themes and categories for focus and elaboration (Strauss, 1987; Wolcott, 1993; Erickson, 1986). Initial open coding procedures were applied to the data from the cohort of methods students in order to identify the larger key themes around which more detailed findings of this study would be explored. For example, initial free-write assignments in the methods course revealed students' general perceptions of elementary mathematics concepts to be "easy." After this general assertion surfaced in the data, further examples and instances in which this topic was manifest in the students' comments and work were categorized, and a more detailed coding procedure was completed in order to understand the subtleties of students' beliefs about elementary mathematics. The data from Julie's case in particular was similarly coded. Specific details from Julie's case illuminated the general themes that surfaced in the findings from the cohort group.

Findings and Interpretations

The findings are organized around two primary themes. The first section deals with the general reactions, beliefs, and perceptions of the cohort group of prospective secondary teachers. This section is followed by a more detailed account of the findings of Julie's case study.

Cohort Findings

Noting differences between elementary and secondary level mathematics. Prior to any explicit engagement in, or class discussion about, elementary mathematics content and pedagogy in the secondary methods courses, the cohort of prospective teachers was asked to respond to a series of writing prompts about elementary level mathematics. There was general agreement within the cohort group that elementary level mathematics was significantly different than secondary level mathematics along several lines. One such prevailing conception was that elementary mathematics was somehow easier, or less complex, than secondary level mathematics.

As one student suggested, "Elementary math is basic. You teach them how to add, subtract, multiply and divide. High school math -- that is where you get into algebra, geometry, precalc/trig, and calculus." Other students made similar comments as they made distinctions between secondary and elementary mathematics. For example, one woman stated that "Secondary math is more abstract, and elementary more concrete. And, of course," she continued, "secondary math has more complex concepts than elementary." Other students suggested that elementary mathematics is devoted to learning the building block "rules," later to be applied and broadened under more difficult circumstances at the secondary level. Said one,

> Elementary math is learning basic skills and rules -- learning the basics and roots of math in order to go on -- whereas secondary math is taking those skills/rules and applying them to learn more complex concepts.... Secondary math is just more complex and abstract. It is tough, and taught at a much faster pace.

Similarly, the students generally made the parallel argument that, since higher level mathematics was more abstract, it was therefore more difficult to teach. As one student stated, "Secondary math is harder to teach. Students are being asked to think and reason in ways they've never seen before."

Doing elementary mathematics. In addition to the writing prompts, the students engaged in a series of activities appropriate for elementary level mathematics. The activities not only required mathematical solutions, but also asked students to think conceptually about their solution strategies. Topics and questions for examination included the following: Why is division by zero undefined? Explain the process of dividing by fractions. Why is the product of two negative numbers a positive number? Why is the slope of a vertical line undefined? Please provide an explanation as to why the traditional long-division algorithm works. How would you model the solution of a two digit multiplication problem? Although the students were able to complete procedures and algorithms related to the concepts at hand without error, they had considerable difficulty offering accompanying conceptual explanations.

For example, one of the questions asked the students to explain or illustrate to a fifth grade student why division by zero was undefined. Only a handful of the students were able to give mathematically sound explanations. Many struggled to find words to explain a rule that they had accepted at face value for so long. One individual, for example, responded by saying that, "I would just tell the student that you can't divide something by nothing." Another student suggested, "Division by zero is undefined because it does not exist." A third noted that "you never get anywhere by adding zeros or multiplying zero by anything." These responses, representative of others made by these prospective secondary teachers, suggest a rather shallow understanding of the concept of division by zero.

A second illustration of similar gaps in the students' conceptual understanding of particular elementary level math concepts revolved around the following question: "Why, when dividing fractions, do you invert the second fraction and multiply? Illustrate your answer as though you were explaining it to an upper elementary or middle school child." Like the previous example, only several of the students presented valid and reasonable explanations, and many left the question blank. The rest of the students typically responded with short, one sentence answers, some of which showed very little conceptual understanding. For example, one students' response included only the following words -- "All division is like dividing fractions." Other students responded to the question by illustrating division of fractions with a sample problem. In general, these students did not offer any written explanation beyond the fractional expression they presented as an example of division of fractions. One such sample of student work was the following: $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times 4 = 2$. Although this math statement is valid, it offers nothing toward the original question -- *why* does one multiply by the reciprocal? As a final example, whether written tongue-in-cheek, or out of frustration, the following remark is nevertheless a good representation of the depth and quality of many of the comments made by students: "If you want to pass this class, accept this fact as true. Always remember to use this fact: Flip and multiply."

Summary of cohort group findings. Revealed through the writings, discussions, and work of the larger cohort of secondary students was a general lack of regard, knowledge, and pedagogical content knowledge concerning elementary level mathematics. The students generally suggested that elementary level mathematics was relatively easy to comprehend and to explain. Implicit in their statements was the notion that, because elementary mathematics comes "before" secondary level mathematics, it must therefore be easier. Despite these broad statements made by nearly all the students in the larger cohort, very few of them were able to give rich explanations for the very concepts that they assumed to be so "easy." Most of the students struggled to give mathematically sound explanations for particular topics commonly studied by elementary and middle school students. Implications of these findings, in light of the case developed below, will be explored in the subsequent discussion.

The case of Julie

The previous description of the cohort's engagements in elementary level mathematics concepts provides a context for a closer look at the case of Julie. Her beliefs about elementary mathematics, descriptions of her experiences in the elementary mathematics pedagogy course, and the impact of that course on her thinking about secondary level mathematics teaching and learning are presented below. I begin by exploring the ways in which Julie recognized limitations in her own mathematical knowledge after working through several engaging activities that might be construed as "elementary" level concepts.

Recognizing deficiencies in her own mathematical knowledge. From the beginning of her experience in the program, Julie was reflective and apprehensive about her rapidly approaching debut as a classroom teacher. In particular, she seemed concerned about her content knowledge. "I know I can't undertake this huge project of understanding all math," she said. "But, day to day, I keep asking, 'Do I have a real understanding of this? How did I learn it? How should I teach it?'" As she pondered these questions repeatedly throughout her preparation experience, she reported frustration in that she did not remember more of her own journey as a mathematics learner both in the k-12 setting, as well as her college level courses. As she recalled about her mathematical knowledge, "I never had a real deep understanding of what is going on. I just learned it for the test. You know -- that is all that math really is. Learn it for the test."

Her frustrations as a learner in mathematics courses led Julie to realize the importance of fostering mathematical understanding in the classroom as a teacher. As the following excerpt indicates, Julie recognized that her own conceptions of mathematics were quite narrow, and that she needed to ensure that she did not perpetuate the same types of experiences for her future students.

> My math knowledge is very by the book. You know, what is in the book is what you need to know. Learning it just to be able to put it down on a test. That is how I learned. What is in the book, and what you need to know in order to get an 'A'. I have learned this past year that that is exactly what I have to avoid when I teach.

Even though Julie's mathematical knowledge was "by the book," and even though she had received excellent grades in her secondary and college level math courses, she became well aware during her field experiences and the elementary mathematics course that she had significant holes in her content knowledge. It became evident to Julie that she did not know mathematics as well as she thought she did, and certainly not as well as she felt teaching required.

> During student aiding, I suddenly realized that I need to know this stuff. I can't go in and fake it....There are a lot of things that I still cannot explain, where I would be apt to say, 'It's just a rule.' And I have to stop myself from saying that-- I realized that I had to be able to explain why things happen.

As explored below, Julie's concerns about her mathematical knowledge contributed to her notable engagement in the elementary methods course.

Perspectives on the preparation coursework. The previous excerpts detail Julie's realization that she had significant holes in her own mathematical knowledge that would likely impact her teaching. In the following paragraphs, Julie's professed deficiencies are explored through the context of her experiences in both the secondary and elementary mathematics methods courses.

Julie shared that the secondary methods courses were just about what she expected them to be. She reported learning a great deal about mathematics curriculum, innovative teaching strategies, and students' learning. As she noted, "I got a very good introduction to issues in mathematics education-- like the *Standards* and reform." She seemed to have a different focus and attitude, however, about the elementary course.

> When I got into the elementary methods, I had a little different perspective [from the secondary methods course]. For one, I knew I already had a teaching job for the next fall, so I wasn't as worried about what I was going to be doing after graduating. So, I could focus a little bit more. I was just trying to see what I could take away from this course, and how it could help me get everything together.

Julie tended to be critical and focused in the elementary course, and she seemed to have a sense of urgency during the course as she was now only a few weeks away from the beginning of her first year as a classroom teacher.

> In there [elementary course], I was really definitely more critical, more interested in how students learn. Because, all the sudden I really needed to learn. I need to know right now-- How do kids learn? What do they have to know? You know -- things like that were always on my mind.

Despite this approach to the course, however, Julie admitted that she initially did not perceive the elementary course as one that was going to challenge her. As she suggested, "I thought the elementary course, honestly, was going to be easy. I thought the math itself that we were going to cover was easy." She realized quickly, however, that it was not going to be as straight forward as she had anticipated.

> Even though I expected it to be easy, it wasn't. I mean, it [the math] was easy to *do*, but is was not easy to understand-- to know what you are doing and why you are doing it. I remember writing on one of those activities you gave us, 'I don't know why I do any of this stuff.' Adding, subtracting... all of it was 'just because.' Just because of the rules. I really never knew why. I never thought deeply into any of this stuff. I still don't think, even after all this, I still don't know fully how or why I do this.

Identified benefits of the elementary mathematics course. That she was challenged in the elementary methods course in ways that did not occur in the secondary preparation courses led Julie naturally to compare the two experiences. The following paragraphs pursue the benefits that Julie identified and attributed to the different kinds of engagement, participation, and ways of thinking that were expected of her in the elementary mathematics course. As she reported, the elementary course not only helped broaden her perspectives on mathematics teaching and learning in general, but also helped her "to see more of my own problems."

One of Julie's initial observations dealt with the differences she observed in the perceptions, attitudes and aptitude of students enrolled in the elementary course. In contrast to the cohort of secondary math students, Julie quickly realized how the prospective elementary teachers lacked the same confidence and many of the math skills that secondary students possessed.

> There was a big difference in the people in the elementary class. There were no math people in the elementary class. I could tell from what they said on the first day that most of these people hated math.

The varying levels of mathematical proficiency between the secondary cohort and the elementary education students led Julie to note several important implications. In particular, Julie recognized right away that once her peers in the elementary class discovered she was a math major, they immediately began to refer to her as the resident math expert. "I was the one they looked to for explanations because they knew I was a math major." Julie recalled. "And I was never quit sure about that." Despite her uneasiness in this role of the expert, Julie recognized afterward that it helped her grow in significant ways. For starters, it required her to think carefully about the mathematical content in ways that did not occur in the secondary course.

> I was the one that was having to think about it. I mean, it is easy to not think about things if you have others in your group that already know it. But I was doing most of the work in my group. In the secondary class, I always tried to engage as much as possible. But, if there was a time when I wasn't quite sure of something, I could back off a little bit until somebody else in my group would say something or help me get back on track. But in the elementary class, I was the expert in the group.... I had to explain my thinking in other ways to these folks, definitely. We all talked about what we were doing and how we were doing it. And, people were just naturally looking to me and how I was doing it.

There was more, however, than Julie simply being forced into doing and explaining much of the mathematics her group encountered. The strategies of her group members, as varied as they often were, allowed her to view mathematics much more broadly. As she continued,

> It was not only that I was the expert, though. How I saw them [group members] doing the math sometimes was interesting. They would approach it sometimes from ways that I originally thought were incorrect -- or at least ways that I would not have chosen to do myself. So, I had to think about that. I ended up engaging more seriously than in the secondary courses. With all the experts in secondary math, we all kind of agreed and all

> kind of understood and went along with each other. But when it got to this level, there were new ideas and some strange ways of thinking about things. There were some interesting ideas that I didn't always agree with, but they got me to engage in it and really think about it.

We see from these excerpts, then, a picture emerging in which Julie found herself engaging in mathematics in a new way. By being thrust into the role of the expert, Julie not only had to learn to communicate her mathematical knowledge clearly, she began to look at mathematics more broadly. This was particularly true as she engaged with peers who often approached the mathematics at hand from non-traditional perspectives. Julie reported that her experiences in the elementary methods course had influenced her thinking about the teaching and learning of mathematics in significant ways. When asked if it had affected her work as a secondary teacher in particular, Julie responded positively. As she noted,

> First, it has changed me not only because I have learned a lot about what I don't know about math, but I have also learned a lot about a whole bunch of issues. First, I definitely won't just say, 'This is a rule.' It will be great for me to be able to look at something and pick it apart and see the elementary or middle school concepts that are a part of the problem. And to be able to help the kids understand it -- to use earlier concepts in what they are doing, and to really understand what it is that they are doing....It was powerful to watch you in the methods course handle a question or topic and be able to explain it in ways other than just a rule. It was interesting for me. I want to be able to do that for my own students. And to give them the power to explain it and understand it themselves.

She also spoke about the ways in which the elementary mathematics course had reinforced concepts that she would be teaching at the secondary level. By dealing with these topics from an elementary level perspective, Julie made discoveries about mathematics. As she described, it was these moments of discovery in the elementary methods class -- her 'aha' experiences -- that were most significant in her development as a beginning secondary teacher.

> What I needed most was more math-- subject by subject. Going through all the subjects [in the elementary course] spending time really looking at math, each part of it from the elementary level on up was very helpful. Those were the best days-- when we were looking at a topic and learning 'Why?' That was definitely the best part of what I got out of the class. The best part about it was engaging in it so that I was having my own 'aha' experiences... Before, I never knew why I was doing what I was doing. You would come around and help us with the manipulatives and it was an amazing 'aha' experience for me. It was just a process of me filling in the holes.

Julie's final reflections. Toward the conclusion of the final interview, Julie reflected on her preparation experience, and on the elementary mathematics course in particular. She recommended that all prospective secondary teachers engage in a similar blend of K-12 mathematics experiences.

> I think that people definitely need to take both the elementary and secondary methods courses. People need to have an understanding about how students learn math. From beginning to end -- knowing where they are coming from.... People need to find out if they really do understand this math. The math courses I took here didn't help me at all towards what I am going to be doing in the classroom.

Julie also spoke more directly about the importance of elementary mathematics for secondary level teaching and learning. As she indicated, the elementary methods course had been fundamental in helping her understand the importance of cultivating good mathematical sense in the early grades.

> I never really understood the importance of elementary mathematics. I mean, middle and secondary math is important. But, at the elementary level is where they learn to think mathematically -- basic math ideas. The first time you deal with this stuff is definitely the most important. By the time they get to secondary level, they have their ways of thinking established from other teachers. That is not true so much at the elementary level. I didn't realize this until I started taking the elementary class.

Discussion and Implications

The title of this article suggests that elementary mathematics may hold some promise in helping prospective secondary teachers develop the content knowledge and pedagogical dispositions to successfully implement reform-based teaching practices at the secondary level. I began the findings of this article by briefly examining the general knowledge base of a cohort of prospective secondary teachers. Echoing what has been revealed previously in the research literature, many of these beginning teachers lacked rich understanding of fundamental mathematical concepts, or were unable to articulate what knowledge they did have. The intent of presenting Julie's case in some detail was to illustrate how explorations in elementary level mathematics and pedagogy -- beyond that which normally occurs in the traditional secondary preparation process -- might address these "missing pieces" in the knowledge and dispositions of secondary teachers. The following discussion highlights several of the implications that emerged from this research.

<u>Recognizing issues of mathematical content knowledge</u>

The revelation that Julie, as well as her cohort of peers in the secondary preparation process, did not possess rich mathematical content knowledge is not surprising given the number of research studies that have revealed similar findings. What is particularly noteworthy about this research, however, is the emergence of considerable evidence to support that Julie recognized these deficiencies in her knowledge, and began to think critically about the impact of her own mathematical understanding on her teaching. As a result of her engagement in the elementary course, Julie reported a growing awareness that her own content knowledge in mathematics was somewhat suspect. Moreover, she began to see how her knowledge structures (or lack thereof) would necessarily impact her teaching at the secondary level. Would she be able to provide rich opportunities for her students to engage in meaningful mathematical experiences if she herself was uncertain about the math at hand?

One apparent implication of this research, then, is that beginning secondary teachers should be encouraged to think seriously about the relationship between teachers' mathematical understanding and their pedagogical practices. Certainly, this kind of examination is not limited to the context of an elementary mathematics course such as the one Julie experienced. The same process may (indeed, should) occur in, among other places, in secondary level methodology courses as well. The point I would like to stress as a result of this research, however, is Julie's repeated suggestion that experiencing elementary level mathematics encouraged her to reflect in a way that did not occur in the secondary methodology courses. For example, recall Julie's indication that her secondary peers tended to quickly agree on traditional conventions and approaches to mathematical concepts without questioning the underlying foundations. In contrast, Julie reported numerous occasions in which she struggled to explain conceptually many of the rules and conventions that she had accepted and used in her advanced study of mathematics for so long. Engaging in the elementary mathematics, it appeared, forced her to think critically and conceptually about many mathematical concepts that were not explored, perhaps even taken for granted, at the secondary level. These findings certainly reflect and support recommendations in the research literature that, if beginning teachers are to teach conceptually, they must have and understand the importance of richly connected knowledge structures.

Recognizing origins and connections among mathematical concepts

In a related sense, the in-depth exploration of elementary-level mathematics concepts and pedagogy revealed a great deal to Julie not only about her own knowledge base, but about how mathematical concepts are connected across the K-12 spectrum. One of the guiding principles of the NCTM *Standards* documents is the notion that learners should grow to appreciate the rich connections that exist among mathematical concepts. As Julie explored fundamental concepts in the elementary methods course, she was able to see connections not only among the topics at hand, but also to some of the mathematical content that she would be responsible to present at the secondary level. As she noted, she began to recognize that a rich understanding of the key, underpinning mathematical concepts was a prerequisite for deeply understanding many secondary level concepts.

Again, arriving at these realizations is certainly not limited to elementary level coursework. Given time necessary to authentically develop and appreciate these connections, however, it seems likely that prospective teachers would benefit from additional experiences beyond those that typically mark the secondary preparation process. As Julie suggested, it was not until she was forced to take the time to explore in the context of an elementary level mathematics course that the idea of "mathematical connections" came to mean something more to her than simply one of many pieces of the reform rhetoric.

Recognizing issues of pedagogy: Diverse solution strategies and approaches

I have suggested previously that the elementary level course Julie experienced is simply one example of a context in which important thinking and development might be encouraged among prospective secondary teachers. Certainly, the important issue is not the elementary level course *itself*, but ways in which the preparation process can be broadened and enhanced through such experiences. The following implication, however, does appear to be necessarily situated within the context of an elementary level course, and provides strong support for promoting experiences such as those Julie encountered.

Recall Julie's reports that her peers in the elementary course treated her as the "math expert." This contributed to Julie's growth in several ways. In her role as the "expert," Julie found herself often taking the lead in her group as she was called upon to by her peers to explain the mathematics at hand. Among other things, these teaching moments helped her see the challenges that await students for whom mathematics does not come easily. She was forced to find multiple ways to articulate and represent mathematical concepts to her peers. The misconceptions they often brought to activities and explorations, as well as the ways those misconceptions could be addressed through the use of hands-on methods and conceptual teaching, were powerful learning moments for Julie. She spoke about how these experiences in the elementary course would help her relate to students in the high school classroom who may be limited in their thinking about particular concepts.

The many unconventional and innovative ways her peers approached problems helped Julie become aware that she needed to recognize, value, and promote mathematics as a process that often allowed for multiple solution strategies and approaches. As Julie reported, despite many opportunities to engage in significant mathematics, these kinds of exchanges simply did not occur in the secondary methods preparation courses. Perhaps the diversity of content knowledge, confidence, and aptitude that were represented in the elementary level course were necessary ingredients to promote Julie's thinking and development in this regard.

Recognizing the importance of the construction of knowledge

Finally, Julie reported that perhaps the most important element of the elementary mathematics experience was that it helped her understand, through her own experiences, how learners construct mathematical understanding. As she engaged in mathematical topic after topic throughout the course, she was making mathematical discoveries along the way. As she suggested, her "Aha!" experiences were filling in the gaps in her own mathematical knowledge. As she worked with manipulatives, viewed the solution strategies of her peers, and tried to find helpful ways to explain mathematical concepts to her fellow group members, she began to discover the "why" behind the rules she had for so long used without thinking. As Julie suggested, being cognizant of the ways in which she was constructing mathematical knowledge throughout the course left an indelible mark on her thinking about teaching.

Again, one needs to ponder the question -- is it important for beginning secondary mathematics teachers to understand the ways in which their future students will (or will not) construct mathematical understanding? If so, then in what ways might this be accomplished in the preparation process? Although Julie believed she understood "constructivism" in a theoretical sense, it was not until she was cognizant of her own constructions of elementary mathematics concepts that she recognized both the significance of student engagement in the learning process, as well as the daunting task of providing such experiences for students. This research suggests, then, that elementary level mathematics coursework might be a fruitful site for prospective secondary teachers to better understand the implications of constructivist theories upon teaching and learning processes.

Conclusion: Broadening the preparation experience

It is an exciting time in mathematics education. Reform visions promoted only a few short years ago continue to generate conversation and receive support. As school mathematics classrooms move away from lecture-dominated environments, the possibilities for creative teaching and deeper mathematical understanding abound. Yet, a generation of beginning teachers continue to be caught in the middle -- being held to high expectations in terms of providing innovative and open instruction, yet having few personal experiences, limited knowledge, and very little of the expertise needed to teach in such a way. A large burden, therefore, rests on mathematics teacher educators. Quite simply, they must find ways to not only enhance prospective teachers' mathematical content knowledge, but also to help them cultivate pedagogical knowledge, beliefs, and teaching practices that will lead to constructivist teaching in the classroom.

Although this study was confined to a relatively small number of students in one teacher preparation program, the findings do hold promise for secondary teacher education. The literature on mathematics pre-service teacher preparation suggests that prospective secondary teachers typically enter education programs with narrow conceptions of mathematics as a set of rules and conventions. The limited content knowledge they possess, as well as the traditional teaching models they often experience in pre-college and college mathematics classes, become obstacles to their acquisition of reform based philosophies and practices. Secondary teacher educators are left with the task of not only introducing students to the vision and methods of reform-based mathematics instruction, but also challenging students' perceptions of "mathematics as rules" and "mathematics teaching as telling" that have been reinforced for years.

This research suggests that there is value in broadening the experiences that typically mark the secondary preparation process. Creating opportunities for secondary students to engage in conceptually based mathematics throughout the preparation experience might be beneficial not only in developing richness in their mathematical content knowledge, but also in their pedagogical content knowledge. Certainly, there are many ways in which prospective teachers, like Julie, may be encouraged to undergo transformations in their thinking about, and understanding of, mathematics teaching and learning. In the case of this research study at hand, the chosen mechanism to experience mathematics in a deeper way was an elementary level mathematics education course. This model appeared to be fairly effective in promoting broader mathematical experiences and understandings that may in fact lead beginning teachers to recognize possibilities for instruction that otherwise likely would have gone undiscovered. As I have tried to illustrate through both the responses of a cohort of secondary teachers as well as the individual case of Julie, secondary teachers can prosper greatly from concentrated examination of, and participation in, elementary mathematics concepts. Perhaps this broader K-12 approach is a missing piece in secondary mathematics teacher preparation.

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